

Energy Distribution of ϕ in Pure Penguin Induced B Decays

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(May, 1995)

Abstract

We study the energy distribution of ϕ in pure penguin induced $B \rightarrow X_s \phi$ taking into account the fermi motion of b inside B meson for $b \rightarrow s\phi$ and also modification due to gluon bremsstrahlung process $b \rightarrow s\phi g$. We find that the contribution to $B \rightarrow X_s \phi$ from $b \rightarrow s\phi g$ is less than 3%. This study provides a criterion for including most of the ϕ 's produced in a penguin process.

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Rare B decays, particularly pure penguin decays, have been a subject of considerable theoretical and experimental interest recently [1]. The photonic penguin processes have been observed by the CLEO collaboration [2] in both the exclusive mode $B \rightarrow K^* \gamma$ and in the inclusive mode $B \rightarrow X_s \gamma$. The Standard Model (SM) is consistent with experimental data [3]. A signature of pure penguin hadronic processes are the exclusive modes $B \rightarrow K \phi$, $K^* \phi$ [4] etc. or the inclusive mode $B \rightarrow X_s \phi$ [5,6] and other modes resulting from processes like $b \rightarrow \bar{s} s s$. The search for exclusive processes has not yet led to a definite observation. The inclusive mode with a larger branching ratio would be a complementary way of searching for penguin processes. At the quark level $B \rightarrow X_s \phi$ results from $b \rightarrow s \phi$, just as $B \rightarrow X_s \gamma$ results from $b \rightarrow s \gamma$. In both cases the energy spectrum of ϕ or γ are not monenergetic as a result of two effects. First, because b quark is in the B meson, its fermi momentum smears the energy spectrum of ϕ or γ . Therefore the distribution of energy depends to some extent on the choice of the wave function. Second effect arises from the process $b \rightarrow s g \gamma$ in the photonic case, and $b \rightarrow s g \phi$ in the hadronic case. The photonic case has been discussed in a series of papers by Ali and Greub [7]. They find that the dominant contribution to the γ spectrum comes from the wave function effect. We shall perform a similar calculation for the hadronic case. The wave function effect is treated with a Monte-Carlo simulation of decays. The gluonic correction is carried out in a simple effective Hamiltonian approximation. We find that the second effect is negligible in our case.

The QCD corrected $H_{\Delta B=1}$ relevant to us can be written as follows [8]:

$$H_{\Delta B=1} = \frac{G_F}{\sqrt{2}} [V_{ub} V_{us}^* (c_1 O_1^u + c_2 O_2^u) + V_{cb} V_{cs}^* (c_1 O_1^c + c_2 O_2^c) - V_{tb} V_{ts}^* \sum c_i O_i] + H.C. , \quad (1)$$

where the Wilson coefficients (WCs) c_i are defined at the scale of $\mu \approx m_b$; and O_i are defined as

$$\begin{aligned} O_1^q &= \bar{s}_\alpha \gamma_\mu (1 - \gamma_5) q_\beta \bar{q}_\beta \gamma^\mu (1 - \gamma_5) b_\alpha , & O_2^q &= \bar{s}_\alpha \gamma_\mu (1 - \gamma_5) q \bar{q} \gamma^\mu (1 - \gamma_5) b , \\ O_{3,5} &= \bar{s}_\alpha \gamma_\mu (1 - \gamma_5) b \sum_{q'} \bar{q}' \gamma_\mu (1 \mp \gamma_5) q' , & Q_{4,6} &= \bar{s}_\alpha \gamma_\mu (1 - \gamma_5) b_\beta \sum_{q'} \bar{q}'_\beta \gamma_\mu (1 \mp \gamma_5) q'_\alpha , \\ O_{7,9} &= \frac{3}{2} \bar{s}_\alpha \gamma_\mu (1 - \gamma_5) b \sum_{q'} e_{q'} \bar{q}' \gamma^\mu (1 \pm \gamma_5) q' , & Q_{8,10} &= \frac{3}{2} \bar{s}_\alpha \gamma_\mu (1 - \gamma_5) b_\beta \sum_{q'} e_{q'} \bar{q}'_\beta \gamma_\mu (1 \pm \gamma_5) q'_\alpha . \end{aligned} \quad (2)$$

The Wilson coefficients at $\mu = m_b$ at the next-to-leading order have been evaluated in Refs. [6,8,9]. For $m_t = 176$ GeV and $\alpha_s(m_Z) = 0.117$, we find

$$\begin{aligned} c_1 &= -0.307, c_2 = 1.147, c_3 = 0.017, c_4 = -0.037, c_5 = 0.010 \\ c_6 &= -0.045, c_7 = 1.2 \times 10^{-5}, c_8 = 3.8 \times 10^{-4}, c_9 = -0.010, c_{10} = 2.1 \times 10^{-3}. \end{aligned} \quad (3)$$

We shall consider the effect due to fermi momentum in $b \rightarrow s\phi$ process, which we assume has the same ϕ energy distribution as $B \rightarrow X_s\phi$, in section (a) below. In section (b) we shall consider the effect due to $b \rightarrow s\phi g$ process.

a. $b \rightarrow s\phi$

Using $H_{\Delta B=1}$ in Eq.(1), we obtain the decay amplitude for $B \rightarrow X_s\phi$

$$A(b \rightarrow s\phi) = -a\bar{s}\gamma_\mu(1 - \gamma_5)b\phi^\mu, \quad (4)$$

where ϵ^μ is the polarization of the ϕ particle; $a = (g_\phi G_F V_{tb} V_{ts}^* / \sqrt{2})[c_3 + c_4 + c_5 + \xi(c_3 + c_4 + c_6) - (c_7 + c_9 + c_{10} + \xi(c_8 + c_9 + c_{10}))]/2]$ with $\xi = 1/N_c$, and N_c is the number of colors. The coupling constant g_ϕ is defined by $\langle \phi | \bar{s}\gamma^\mu s | 0 \rangle = ig_\phi \epsilon^\mu$. From the experimental value for $Br(\phi \rightarrow e^+e^-)$ [10], we obtain $g_\phi^2 = 0.0586 \text{ GeV}^4$. The branching ratio for $b \rightarrow s\phi$ is predicted to be 1.7×10^{-4} [6] for $\alpha_s(m_Z) = 0.117$.

The decay rate is given by

$$\Gamma(b \rightarrow s\phi) = \frac{|a|^2 m_b^3}{8\pi m_\phi^2} \lambda_{s\phi}^{3/2} \left[1 + \frac{3}{\lambda_{s\phi}} \frac{m_\phi^2}{m_b^2} \left(1 - \frac{m_\phi^2}{m_b^2} + \frac{m_s^2}{m_b^2} \right) \right], \quad (5)$$

where $\lambda_{ij} = (1 - m_j^2/m_b^2 - m_i^2/m_b^2)^2 - 4m_i^2 m_j^2 / m_b^4$.

To study the energy distribution of ϕ , we adopt the model in Ref. [11] in which the b quark is not at rest inside B but with a fermi momentum p_b according to a Gaussian distribution,

$$\Phi(\vec{p}_b) = \frac{4}{\sqrt{\pi} p_f^3} e^{-\vec{p}_b^2 / p_f^2}, \quad (6)$$

where the parameter p_f is determined from experimental data to be between 0.21 to 0.39 GeV [12]. The b quark mass expressed in terms of the B meson mass m_B and the spectator quark mass m_q in the rest frame of B , is given by

$$m_b^2 = m_B^2 + m_q^2 - 2m_B\sqrt{\vec{p}_b^2 + m_q^2}. \quad (7)$$

In the rest frame of the B , the $b \rightarrow s\phi$ decay width $\Gamma(m_b)$ is given by $(m_b/E_b)\Gamma(b \rightarrow s\phi)$. Its contribution to the decay width $\Gamma_{s\phi}$ for $B \rightarrow X_s\phi$ is averaged over all allowed momenta \vec{p}_b . We have

$$\begin{aligned} \Gamma_{s\phi}(B \rightarrow X_s\phi) &= \int_0^{P_{max}} \Phi(\vec{p}_b) \vec{p}_b^2 \Gamma(m_b) d|\vec{p}_b|, \\ P_{max} &= \sqrt{\frac{(m_B^2 + m_q^2 - (m_\phi + m_s)^2)^2}{4m_B^2} - m_q^2}. \end{aligned} \quad (8)$$

Due to the finite momentum distribution, the energy of ϕ from b quark decay is no longer monoenergetic, instead there will be a distribution. The ϕ energy spectrum generated from a Monte-Carlo simulation of the decay is shown in Figure 1. In the figures we have used the constituent mass of 0.3 GeV and 0.5 GeV for the spectator quark and the s -quark, respectively. From Fig.1, we see indeed that there is a spread in the ϕ energy with the maximum located at about 2.55 GeV.

b. $b \rightarrow s\phi g$

The quark level effective Hamiltonian responsible for $b \rightarrow s\phi g$ is complicated. We use the simplified effective Hamiltonian in Eq.(4) to obtain the ϕ energy distribution for the process $b \rightarrow s\phi g$ by attaching a gluon on either the initial b or the final s quarks. This is expected to be a good approximation because the dominant effect comes from the bremsstrahlung of gluon emission from the external light quark. We obtain

$$\begin{aligned} A(b \rightarrow s\phi g) &= ag_s \left(\frac{\bar{s}\gamma_\mu(2p_b^\nu - \not{p}_g\gamma^\nu)T^a(1 - \gamma_5)b}{(p_b - p_g)^2 - m_b^2} \right. \\ &\quad \left. + \frac{\bar{s}(2p_s^\nu + \gamma^\nu \not{p}_g)\gamma_\mu T^a(1 - \gamma_5)b}{(p_s + p_g)^2 - m_s^2} \right) G_\nu^a \phi^\mu, \end{aligned} \quad (9)$$

where p_b , p_s , and p_g are the b-quark, s-quark and gluon momenta, respectively. From above we have the following ϕ energy spectrum

$$\begin{aligned} \frac{d\Gamma(b \rightarrow s\phi g)}{dE_\phi} &= \frac{|a|^2 \alpha_s}{32\pi^2 m_b^2} \frac{N_c^2 - 1}{N_c^2} \int_{t_{min}}^{t_{max}} dt \left[\frac{1}{1+Y} (4 + (1+Y)^2 + (1-Y)^2 \frac{1+\mu_s^2}{2\mu_\phi^2}) \right. \\ &\quad \left. + \frac{2}{X^2(1+Y)} (1 - 2\mu_\phi^2 - X - Y + XY - 2\mu_s^2 \frac{1-Y}{1+Y}) \left(\frac{(1-\mu_s^2)^2}{\mu_\phi^2} - 2\mu_\phi^2 + 1 + \mu_s^2 \right) \right], \end{aligned} \quad (10)$$

where

$$\begin{aligned} X &= \frac{s+t-m_s^2-m_\phi^2}{m_b^2}, \quad Y = \frac{s+t-m_s^2-m_\phi^2}{s-t-m_s^2+m_\phi^2}, \\ s &= m_b^2 + m_\phi^2 - 2m_b E_\phi, \quad \mu_{s,\phi} = \frac{m_{s,\phi}}{m_b}, \\ t_{max,min} &= \frac{(s-m_s^2)}{2s} ((m_b^2 - s - m_\phi^2) \pm \sqrt{(m_b^2 - s - m_\phi^2)^2 - 4m_\phi^2 s}) + m_\phi^2. \end{aligned} \quad (11)$$

The energy distribution in Eq.(10) has the well-known infrared divergence due to the zero mass of the gluon. To regulate the infrared divergence, we assign an effective gluon mass of about $2m_\pi$ which represents the lowest invariant mass of the gluon. The ϕ energy distribution for $b \rightarrow s\phi g$ is shown in Figure 2. Here we have neglected the effect due to non-zero \vec{p}_b discussed in the previous section which is small and approximated the $b \rightarrow s\phi g$ contribution to $B \rightarrow X_s \phi$ by Eq.(10). We find that the effect of $b \rightarrow s\phi g$ on $B \rightarrow X_s \phi$ is small because $BR(b \rightarrow s\phi g)/BR(b \rightarrow s\phi)$ is only about 3%. The total energy distribution is shown in Figure 3.

The spectrum of ϕ should approximate the spectrum that arises from the decays $B \rightarrow K\pi\phi$, $K\pi\pi\phi$, etc. Of course the monoenergetic ϕ 's that arise from two body modes like $B \rightarrow K\phi$ or $K^*\phi$ are included in an average sense. The specific two body modes are not expected to be more than 10% of the inclusive $X_s \phi$ production [6]. In this paper we have not discussed the ϕ energy spectrum from the decay of the dominant non-penguin processes which are expected to have a much softer spectrum since they always arise from decay of charmed states. We assume that this experimentally well known contribution has been subtracted in the region of interest. If in addition to the selection criterion on ϕ discussed in this letter, X_s will be experimentally shown to include an odd number of kaons, then the penguin process will be even more enhanced [4].

ACKNOWLEDGMENTS

NGD and XGH thank Drs. Z. Chao, R. Frey and D. Strom for helps in computer simulations. NGD, XGH and JT were supported in part by the Department of Energy Grant No. DE-FG06-85ER40224. GE was supported in part by GIF-The German-Israeli Foundation for Scientific Research and Development and by the New York Metropolitan Research Fund.

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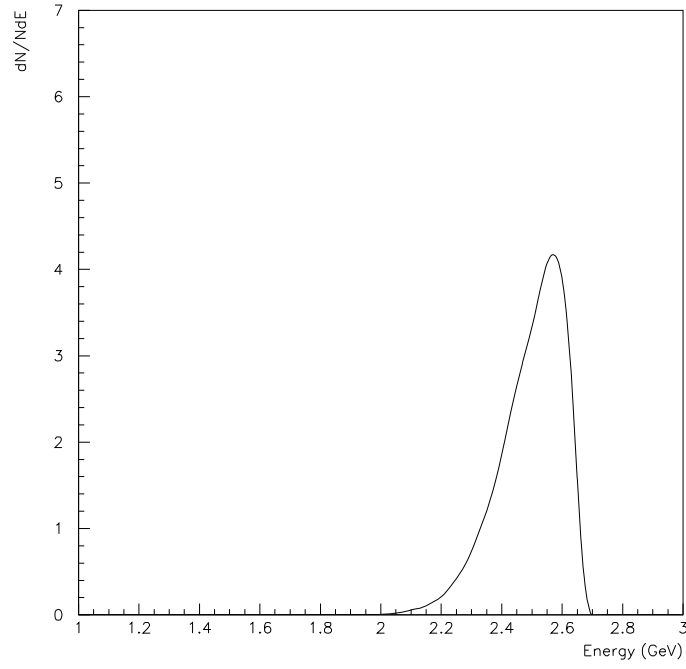


FIG. 1. E_ϕ distribution for $b \rightarrow s\phi$ for $p_f = 0.3$ GeV.

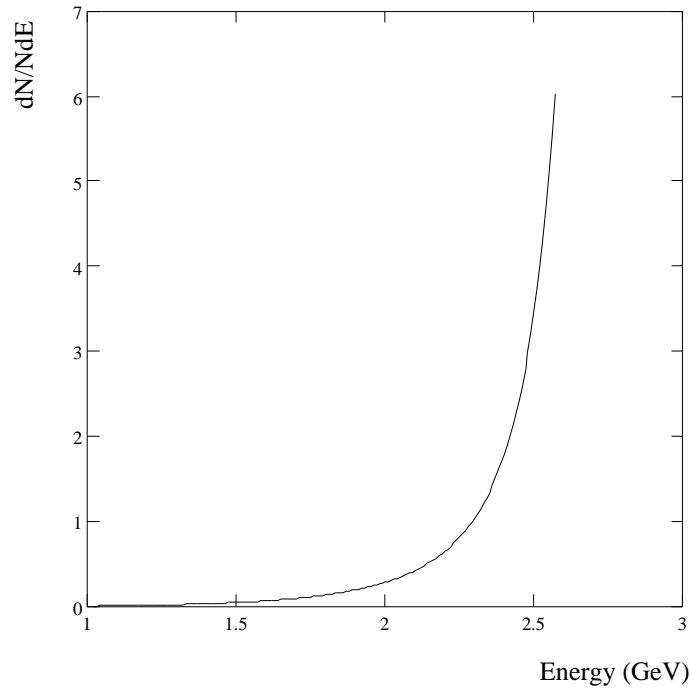


FIG. 2. E_ϕ distribution for $b \rightarrow s\phi g$.

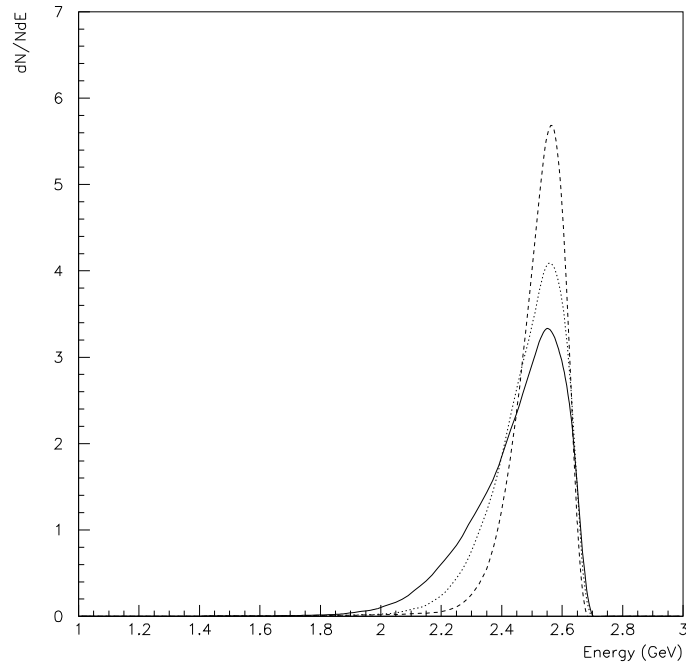


FIG. 3. E_ϕ distribution for $b \rightarrow s\phi + s\phi g$. The solid, dotted and dashed lines are for $p_f = 0.39$ GeV, 0.30 GeV and 0.21 GeV, respectively.